Transformations of the graphs of functions
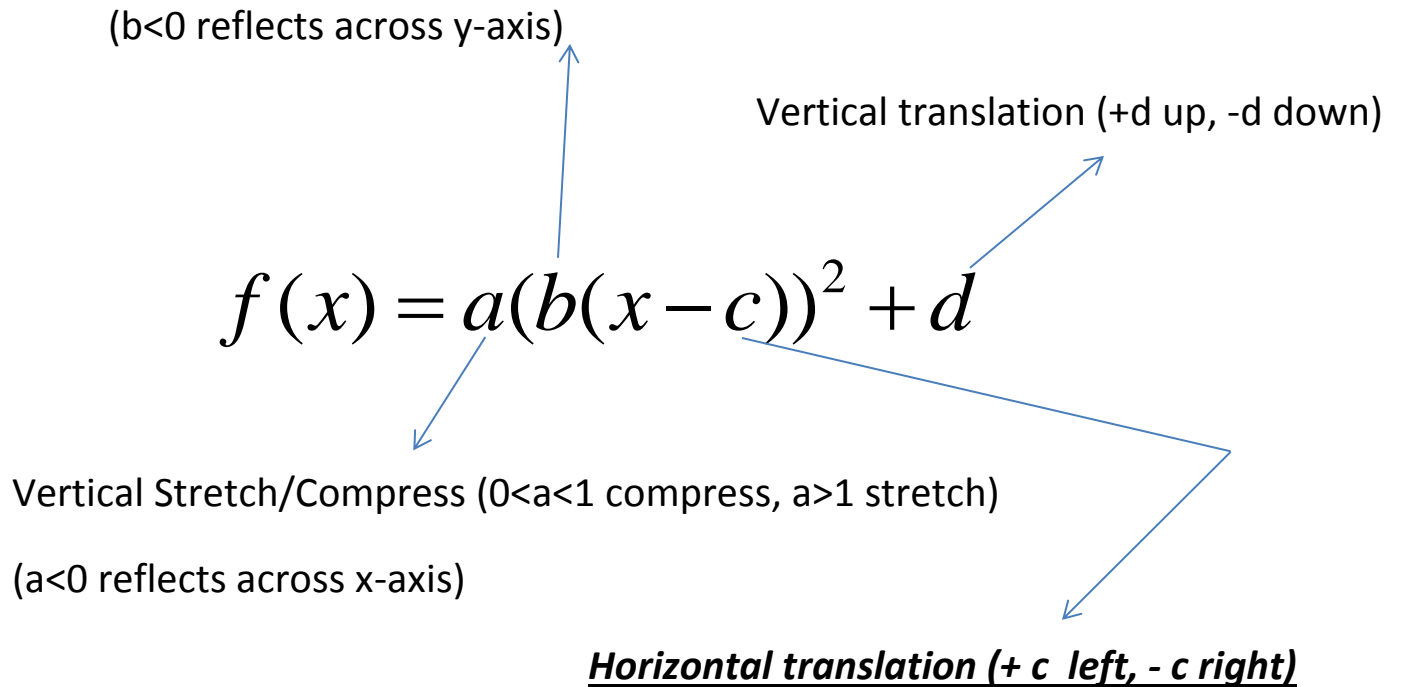
	Notation	In words	Sample Parent function	Transformation	In words	Graph Transformation
Translations	$f(x)+d$	Translate $f(x)$ up d units $(d>0)$	$f(x)=x^2$	$f(x)=x^2+2$	$f(x)=x^2$ moves up 2 units	
	$f(x)-d$	Translate $f(x)$ down c units $(d>0)$	$f(x)=\dfrac{1}{x}$	$f(x)=\dfrac{1}{x}-2$	$f(x)=\dfrac{1}{x}$ moves down 2 units	
	$f(x+c)$	Translate $f(x)$ left c units $(c>0)$	$f(x)=\dfrac{1}{x^2}$	$f(x)=\dfrac{1}{(x+3)^2}$	$f(x)=\dfrac{1}{x^2}$ moves left 3 units	
	$f(x-c)$	Translate $f(x)$ right c units $(c>0)$	$f(x)=\sqrt{x}$	$f(x)=\sqrt{x-1}$	$f(x)=\sqrt{x}$ moves right 1 unit	
Reflections	$-f(x)$	Reflect $f(x)$ across the x-axis	$f(x)=2^x$	$f(x)=-\left(2^x\right)$	$f(x)=2^x$ reflects across the x-axis	
	$f(-x)$	Reflect $f(x)$ across the y-axis	$f(x)=\ln x$	$f(x)=\ln(-x)$	$f(x)=\ln x$ reflects across the y-axis	
Stretches/Compressions	$af(x)$	If a > 1: Stretches $f(x)$ vertically by factor of a	$f(x)=\lvert x\rvert$	$f(x)=2\lvert x\rvert$	$f(x)=\lvert x\rvert$ stretches vertically 2 times taller	
		If 0 < a < 1: Compresses $f(x)$ vertically by factor of a	$f(x)=\sqrt{x}$	$f(x)=\dfrac{1}{3}\sqrt{x}$	$f(x)=\sqrt{x}$ compresses vertically $\dfrac{1}{3}$ times smaller	
	$f(bx)$	If b > 1: Compresses $f(x)$ horizontally by factor of $\dfrac{1}{b}$	$f(x)=x^3$	$f(x)=(2x)^3$	$f(x)=x^3$ compresses horizontally $\dfrac{1}{2}$ times smaller	
		If 0 < b < 1: Stretches $f(x)$ horizontally by factor of $\dfrac{1}{b}$	$f(x)=\sqrt[3]{x}$	$f(x)=\sqrt[3]{\dfrac{1}{4}x}$	$f(x)=\sqrt[3]{x}$ stretches horizontally 4 times wider	

Here is a look at the parent function $f(x) = x^2$ **with all components**

Horizontal Stretch/Compress (b>1 compress, 0<b<1 stretch)

(b<0 reflects across y-axis)

Vertical translation (+d up, -d down)

$$f(x) = a(b(x-c))^2 + d$$

Vertical Stretch/Compress (0<a<1 compress, a>1 stretch)
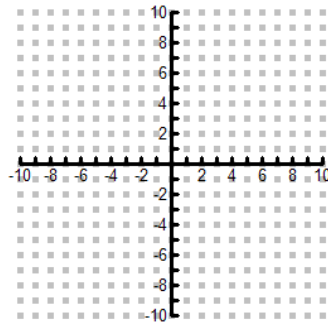
(a<0 reflects across x-axis)

Horizontal translation (+ c left, - c right)

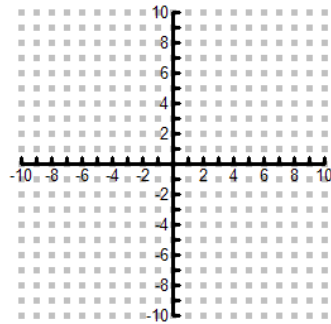# You should NOT need a calculator for any of the following questions.

First we'll just focus on the translations.

For each of the following problems, name the parent function, describe the transformations, then sketch its graph.
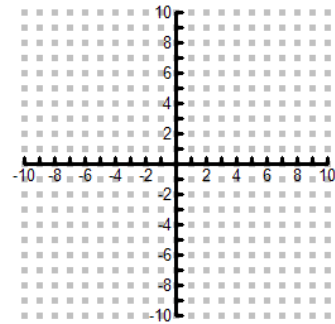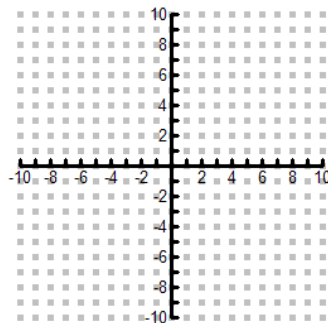
1. $f(x) = \sqrt{x+3}$
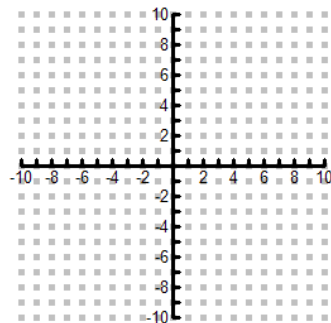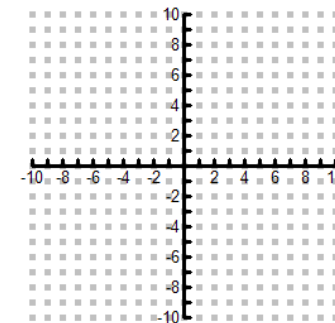
2. $f(x) = x^2 - 2$

3. $f(x) = (x-4)^3$

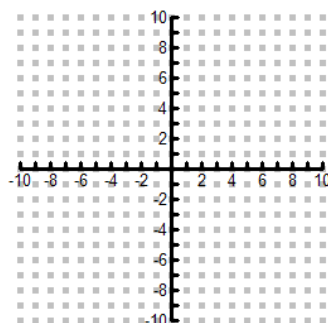4. $f(x) = \dfrac{1}{x-5}$

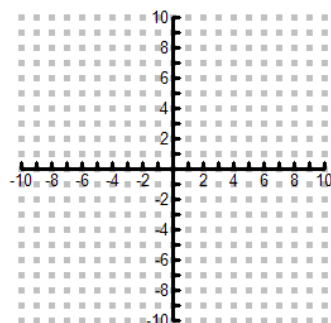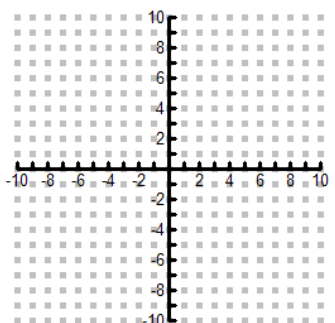5. $f(x) = \ln x + 4$

6. $f(x) = e^{x+1}$

7. $f(x) = |x+1| - 2$

8. $f(x) = (x-7)^2 - 4$

9. $f(x) = \sqrt[3]{x-5} + 3$

Given the description and the parent function, write the equation, then state the domain and range for each.

1. $f(x) = \sqrt{x}$ translated right 4 and down 7
   - Equation:
   - Domain:
   - Range:

2. $f(x) = \ln x$ translated right 2
   - Equation:
   - Domain:
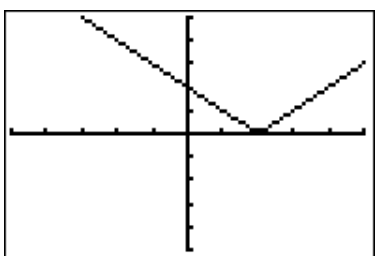   - Range:

3. $f(x) = x^3$ translated left 2 and up 9
   - Equation:
   - Domain:
   - Range:

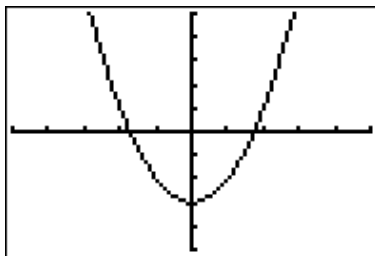4. $f(x) = e^x$ translated left 8 and down 1
   - Equation:
   - Domain:
   - Range:

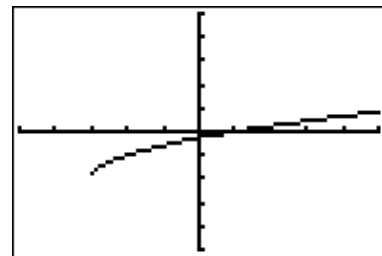Write the function that would make each graph. (The x-scale and y-scale is 1).
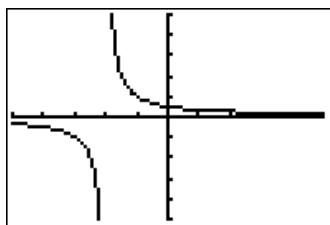
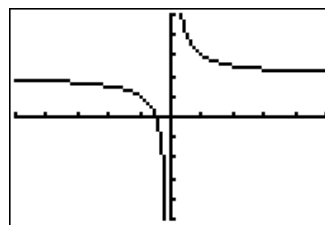5.

6.

7.

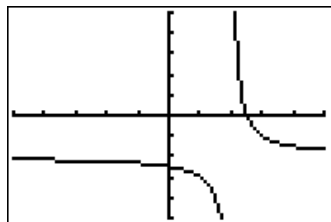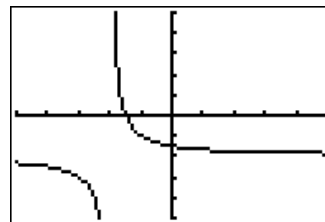Match each equation with its corresponding graph.

A.

B.

C.

D.

8. $f(x) = \dfrac{1}{x-2} - 2$
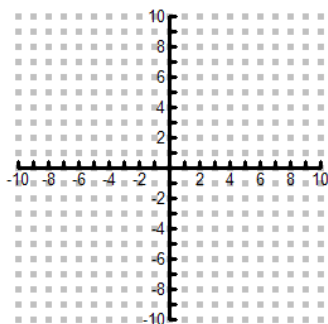
9. $f(x) = \dfrac{1}{x+2} - 2$

10. $f(x) = \dfrac{1}{x} + 2$
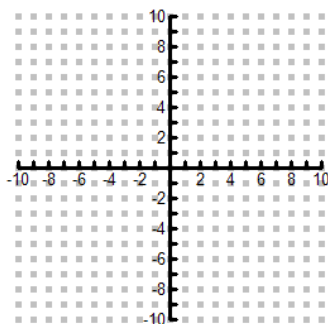
11. $f(x) = \dfrac{1}{x+2}$

Now let's focus on just reflections.

For each of the following problems, name the parent function, describe the transformations, then sketch its graph.
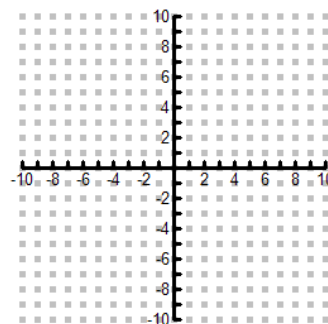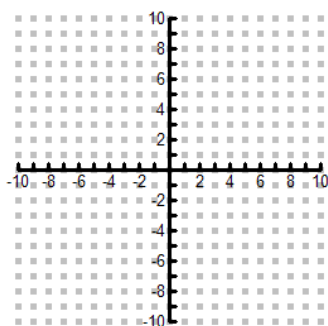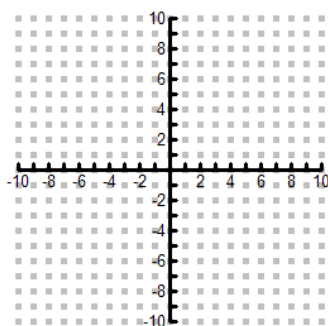
1. $f(x) = -\sqrt{x}$
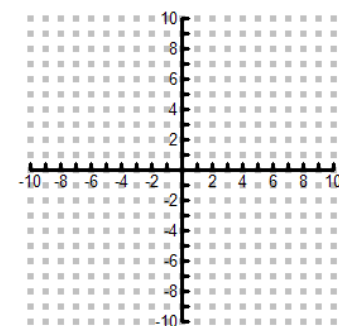
2. $f(x) = \sqrt{-x}$

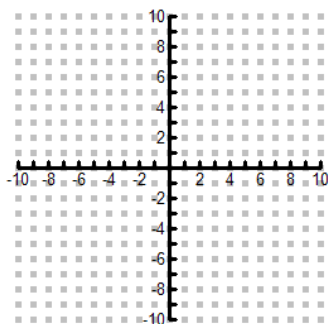3. $f(x) = -\dfrac{1}{x^2}$

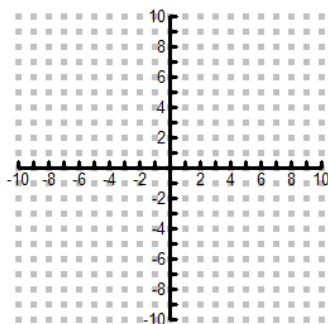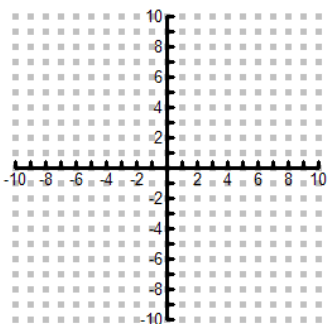4. $f(x) = \dfrac{1}{(-x)^2}$

5. $f(x) = \ln(-x)$

6. $f(x) = \left| -x \right|$

7. $f(x) = -\left| x \right|$

8. $f(x) = -e^{-x}$

9. $f(x) = -\sqrt[3]{-x}$

Given the description and the parent function, write the equation, then state the domain and range for each.

1. $f(x) = x^3$ reflected across the x-axis
   - Equation:
   - Domain:
   - Range:

3. $f(x) = \dfrac{1}{x}$ reflected across the y-axis.
   - Equation:
   - Domain:
   - Range:

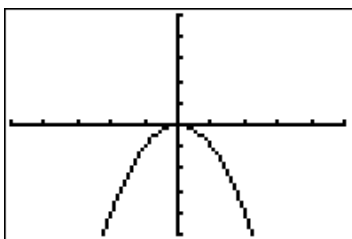2. $f(x) = x^2$ reflected across both axes
   - Equation:
   - Domain:
   - Range:

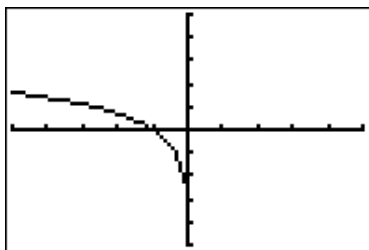4. 4. $f(x) = e^x$ reflected across the x-axis
   - Equation:
   - Domain:
   - Range:

Write the function that would make each graph. (The x-scale and y-scale is 1)
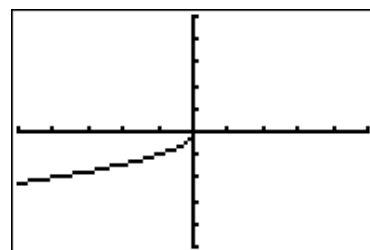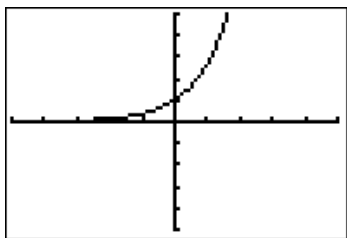
5.
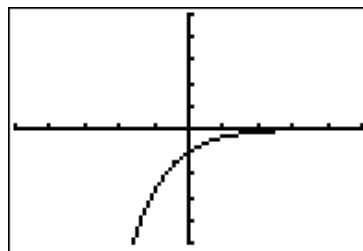
6.

7.

Match each equation with its corresponding graph.
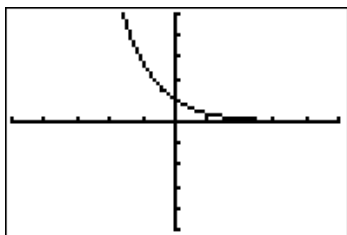
A.

B.

C.

D.

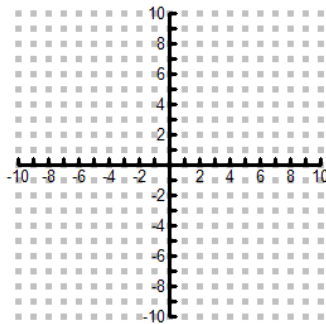8. $f(x) = e^{-x}$ 9. $f(x) = -e^{-x}$ 10. $f(x) = -e^x$ 11. $f(x) = e^x$
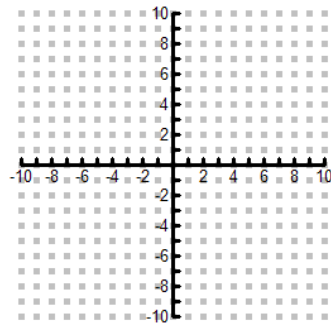
Now we'll focus on just stretches and compressions.

For each of the following problems, name the parent function, describe the transformations, then sketch its graph.
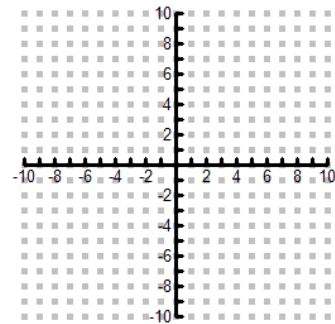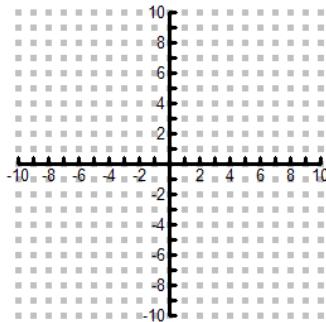
1. $f(x) = \dfrac{1}{2}|x|$
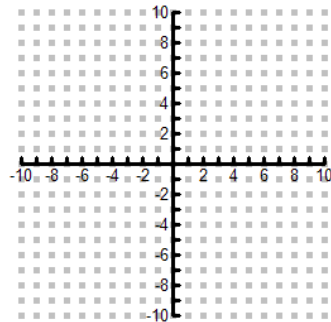
2. $f(x) = |2x|$

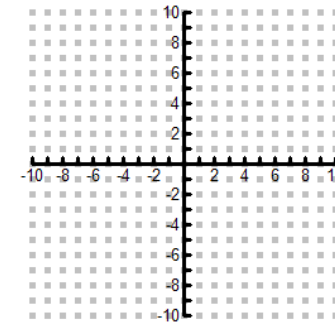3. $f(x) = \left(\dfrac{1}{3}x\right)^3$

4. $f(x) = 3x^3$

5. $f(x) = 3\sqrt{x}$

6. $f(x) = \sqrt{\dfrac{1}{2}x}$

For each of the following problems, name the parent function, then describe the transformations.

7. $f(x) = 2(3x)^2$

8. $f(x) = 2\sqrt{\dfrac{x}{3}}$

9. $f(x) = \dfrac{1}{2}(3x)^2$

Given the description and the parent function, write the equation, then state the domain and range for each.

1. $f(x) = \sqrt{x}$ vertically stretched by factor of 3.
    - Equation:
    - Domain:
    - Range:

2. $f(x) = \ln x$ horizontally stretched by factor of 3.
    - Equation:
    - Domain:
    - Range:

3. $f(x) = x^3$ vertically compressed by factor of ½ and horizontally stretched by factor of 3.
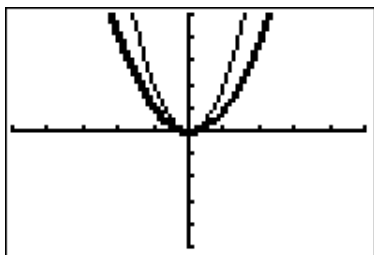   - Equation:
    - Domain:
    - Range:

4. $f(x) = e^x$ vertically stretched by factor of 2 and horizontally compressed by factor of ¼.
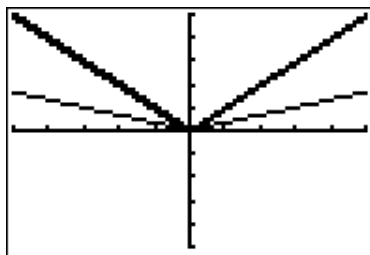   - Equation:
    - Domain:
    - Range:

Write the function that would make each graph. (The x-scale and y-scale is 1, the bold graph is the parent function).
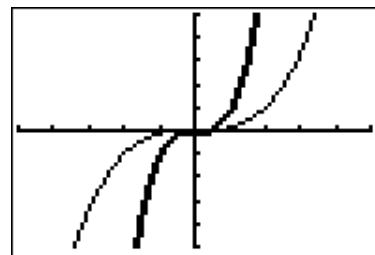
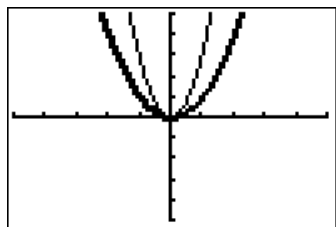5. (vertical stretch of 2)   6. (vertical compression of 1/3)   7. (horizontal stretch of 2)
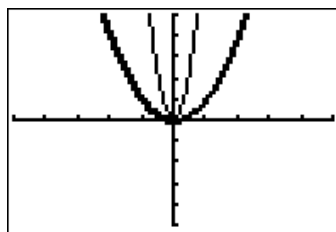

Match each equation with its corresponding graph. (bold: y = x²)
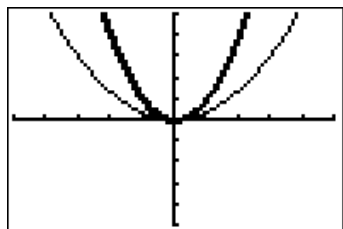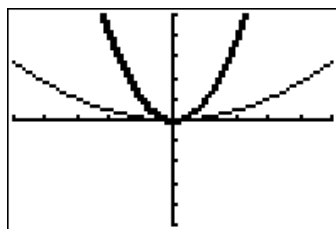
A.

B.

C.

D.

8. $f(x) = 3x^2$ 9. $f(x) = \dfrac{1}{3}x^2$ 10. $f(x) = \left(\dfrac{1}{3}x\right)^2$ 11. $f(x) = (3x)^2$

We are now going to look at functions that have a mixture of transformations.

Here is a look at the parent function $f(x) = x^2$ with all components

Horizontal Stretch/Compress (b>1 compress, 0<b<1 stretch)
(b<0 reflects across y-axis)

Vertical translation (+d up, -d down)

$$f(x) = a(b(x-c))^2 + d$$

Vertical Stretch/Compress (0<a<1 compress, a>1 stretch)

(a<0 reflects across x-axis)

Horizontal translation (+ c left, - c right)

Steps for Multiple Transformations:

Use the following order to graph a function involving more than one transformation:

1. Stretching/Compressing
2. Reflecting
3. Translations

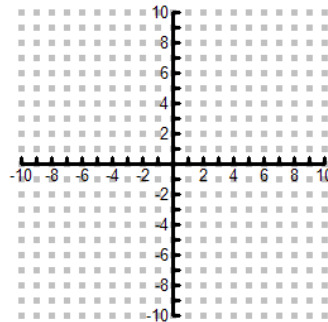## You should NOT need a calculator for any of the following questions.

We will now focus on problems that have a mix of transformations.

For each of the following problems, name the parent function, describe the transformations in the correct order in which they would be performed, then sketch its graph.
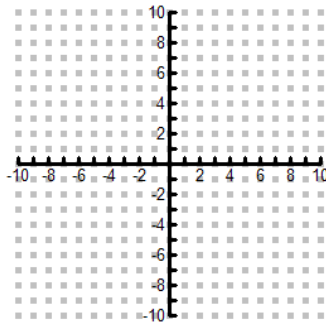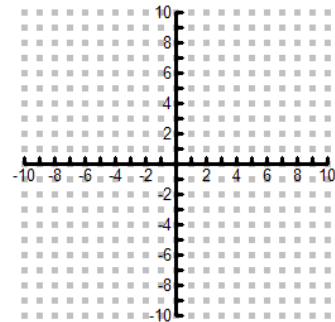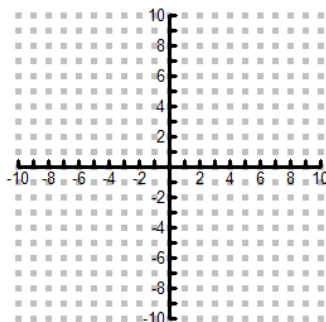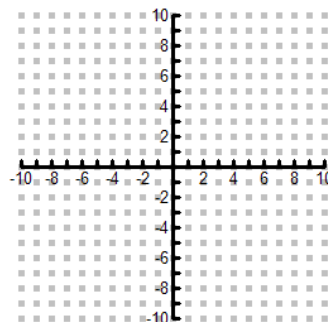
1. $f(x) = -\sqrt{x+2}$

2. $f(x) = 2(x+2)^2$

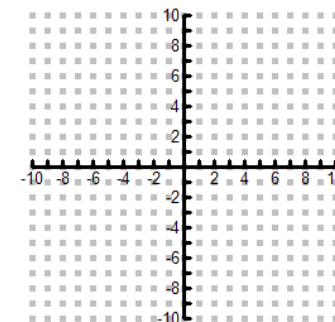3. $f(x) = \left(\dfrac{1}{2}x\right)^3 + 4$
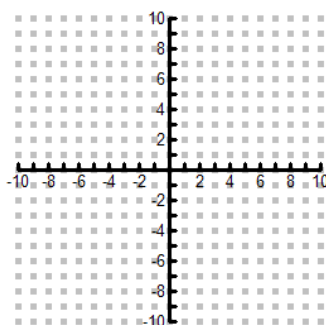
4. $f(x) = -\dfrac{1}{x^2} + 2$
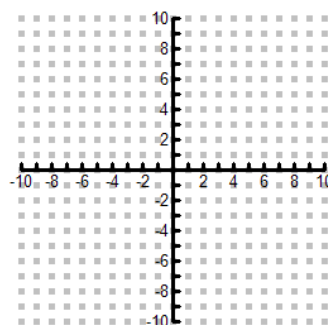
5. $f(x) = -\ln(x+1)$

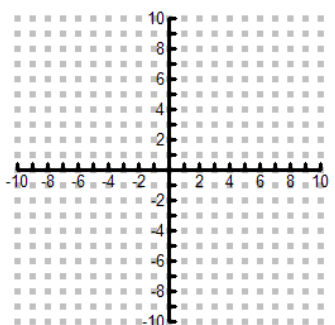6. $f(x) = \left|\dfrac{1}{3}(x-2)\right|$

7. $f(x) = -|x+3| - 2$

8. $f(x) = 2(e)^{-x} - 1$

9. $f(x) = \sqrt[3]{2(x-5)} - 4$

Given the description and the parent function, write the equation.

1. $f(x) = x^3$ reflected across the y-axis,
   then translated 2 units down.
   - Equation:

3. $f(x) = \dfrac{1}{x}$ vertically stretched by factor of 3,
   then translated 3 units left .
   - Equation:

2. $f(x) = x^2$ horizontally stretched by factor
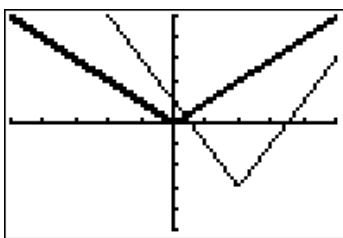   of 2, then reflected across the x-axis.
   - Equation:

4. $f(x) = e^x$ horizontally compressed by
   factor of $\dfrac{1}{4}$, then reflected across the y-axis
   translated 2 units right, then 5 units up.
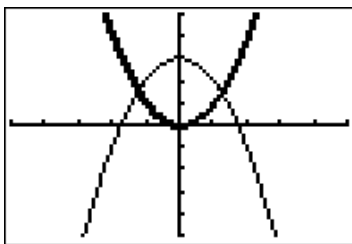   - Equation:

Write the function that would make each graph. (The x-scale and y-scale is 1, the bold graph is the parent function).
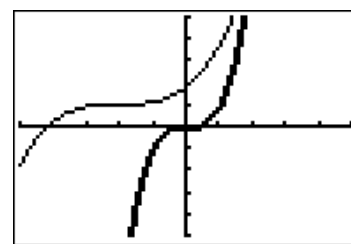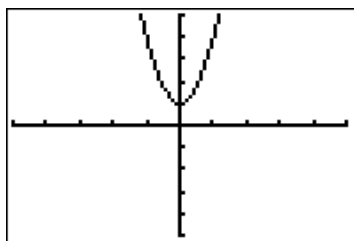
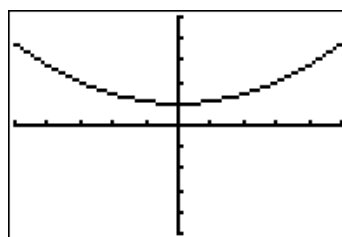5.   6.   7. (hint: there's a horizontal stretch of 2)

Match each equation with its corresponding graph

A.
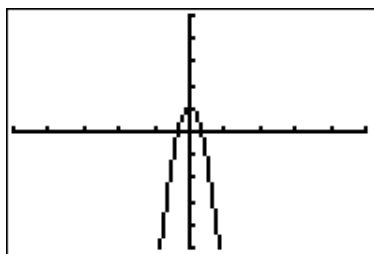

B.
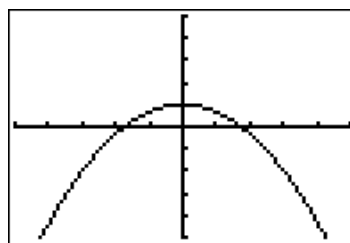

C.

D.

8. $f(x) = 3x^2 + 1$ 9. $f(x) = \left(\dfrac{1}{3}x\right)^2 + 1$ 10. $f(x) = -(3x)^2 + 1$ 11. $f(x) = -\dfrac{1}{3}x^2 + 1$